RG Oscillatory Boundary Layers

7/29/2019

①

1st Neglect Earth's rotation

$$\frac{\partial u}{\partial t} - \frac{1}{\rho}\frac{\partial \tau}{\partial z} = -g\frac{\partial \eta}{\partial x}$$

$$-i\omega\mathcal{U} - A\,\mathcal{U}'' = -i\omega\,\mathcal{U}_\infty$$

Invoke simple turbulence closure

$$\frac{\tau}{\rho_0} = A\frac{\partial u}{\partial z} \quad \text{where } A \text{ is } \underline{\text{constant}}$$

Now consider oscillatory solution

$$u(z,t) = \mathrm{Re}\{\mathcal{U}(z)\,e^{-i\omega t}\}$$

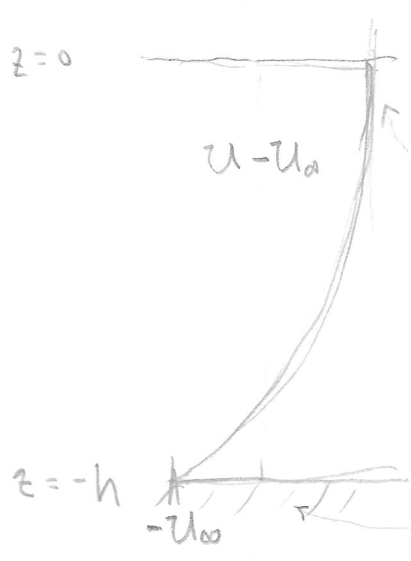
$$u|_\infty = \mathcal{U}_\infty e^{-i\omega t} \sim u(A=0)$$

define $\mathcal{U}_\infty$ such that $-g\frac{\partial \eta}{\partial x} = -i\omega\,\mathcal{U}_\infty e^{-i\omega t}$

$$\mathcal{U}'' + \frac{i\omega}{A}(\mathcal{U} - \mathcal{U}_\infty) = 0$$

$$\sim (\mathcal{U} - \mathcal{U}_\infty)'' + \frac{i\omega}{A}(\mathcal{U} - \mathcal{U}_\infty) = 0$$

$$\mathcal{U} - \mathcal{U}_\infty = a\cos kz + \underset{0}{b}\sin kz \qquad \text{where } k = \sqrt{i\frac{\omega}{A}}$$



$$\frac{\partial}{\partial z}(\mathcal{U} - \mathcal{U}_\infty)\Big|_{z=0} = 0 \qquad \text{no stress at surface}$$

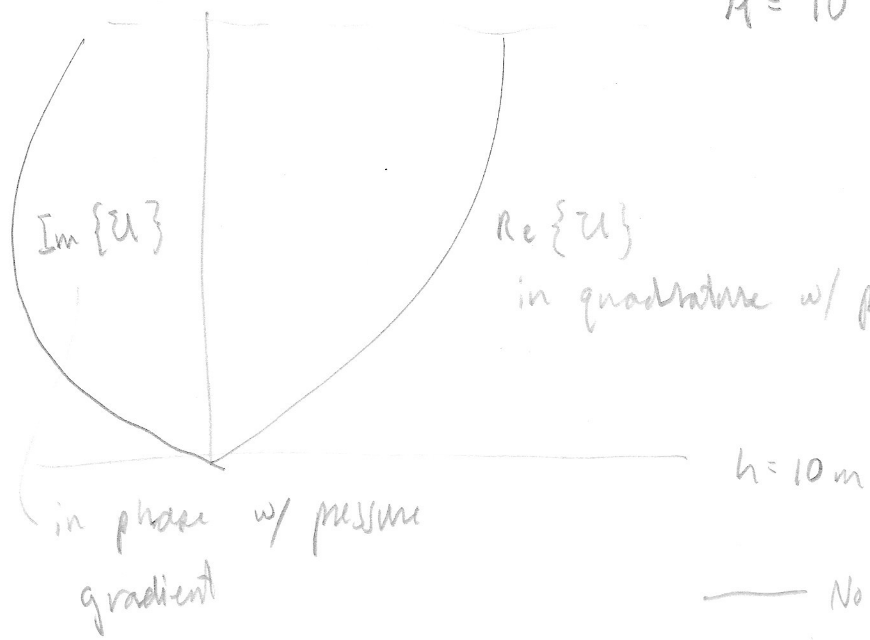
$$(\mathcal{U} - \mathcal{U}_\infty)\Big|_{z=-h} = -\mathcal{U}_\infty \qquad \text{because } \mathcal{U}(-h) = 0$$

$$\Rightarrow \quad a\cos(kh) = -u_\infty$$

but k is complex

$$a = \frac{-u_\infty}{\cos kh} \quad \text{so} \quad u = u_\infty\left(1 - \frac{\cos kz}{\cos kh}\right)$$

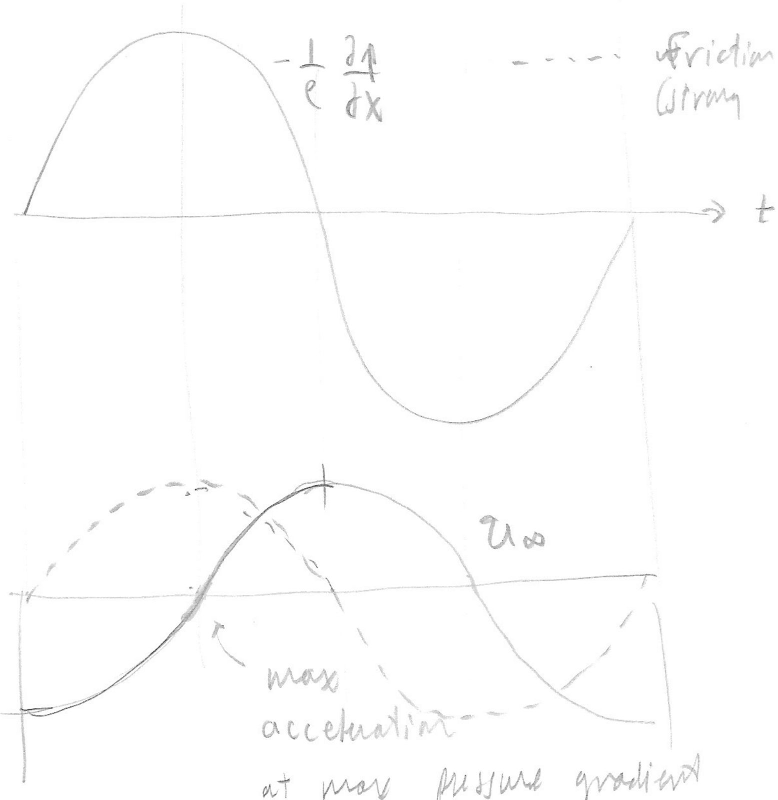
$$A = 10^{-3}\ m^2/s$$



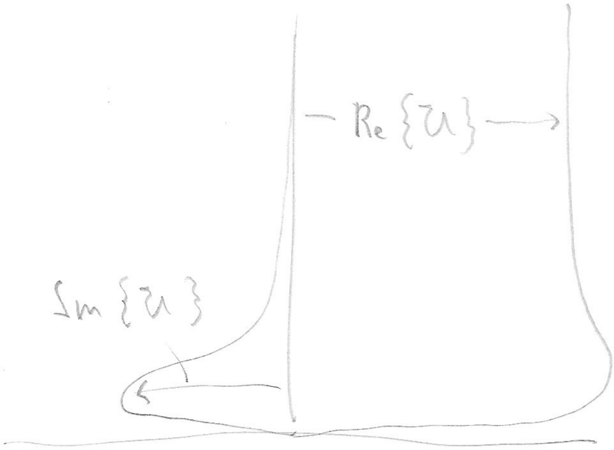
—— No friction $\frac{\partial u_\infty}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$

- - - - Friction (strong) $\cancel{\frac{\partial u_\infty}{\partial t}} + \text{friction} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$

so friction shifts things earlier (compared to just acceleration



With $A = 10^{-4}\ m^2/s$



Now let's turn on Coriolis!

$$u_t - fv - A u_{zz} = -g\eta_x$$

$$v_t + fu - A v_{zz} = -g\eta_y$$

$$\Rightarrow \quad -i\omega U - fV - AU'' = -\frac{1}{\rho} p_x \quad (1)$$

$$-i\omega V + fU - AV'' = -\frac{1}{\rho} p_y \quad (2)$$

Multiply (2) by i and add to (1)

$$-i\omega (U + iV) \overset{*}{+} if(U + iV) - A(U + iV)'' = -\frac{1}{\rho}(p_x + i p_y)$$

$$-i(\omega \overset{*}{-} f)(U + iV) - A(U + iV)'' = -\frac{1}{\rho}(p_x + i p_y)$$

$*$ = check sign

two solutions

$W_1 = U + iV$ cyclonic (counter clockwise)

$W_2 = U - iV$ anticyclonic (clockwise)

What we did.

$$k_1 = \left[\frac{i(\omega+f)}{A} \right]^{\frac{1}{2}}$$

$\omega_{M_2} = 1.4 \times 10^{-4}\ s^{-1}$

$f = 1 \times 10^{-4}\ s^{-1}$

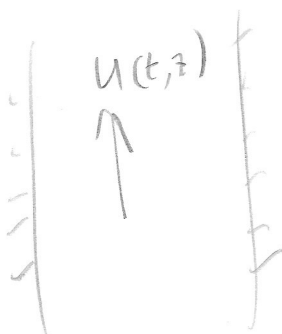
$$k_2 = \left[\frac{i(\omega-f)}{A} \right]^{\frac{1}{2}}$$

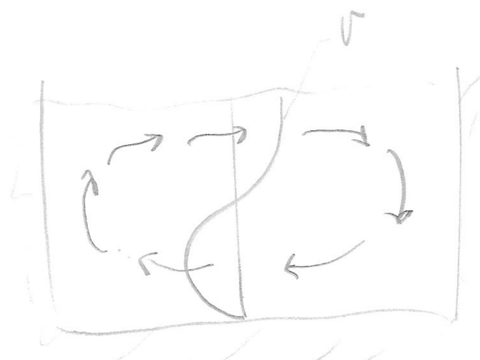
$$\left.\begin{array}{l} \omega + f = 2.4 \times 10^{-4} \\ \omega - f = 0.4 \times 10^{-4} \end{array}\right\} \Rightarrow \text{very different } k\text{'s !}$$

$$\frac{W_1 + W_2}{2} = U$$

$$\frac{W_1 - W_2}{2} = V$$

← in a long channel $\int_{-h}^{0} V\, dz = 0$





an example of secondary circulation